SAFE HANDS & IIT-ian's PACE

LEAP TEST# 15 (JEE) ANS KEY Dt. 10-01-2024

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	D	31	B	61	A
2	D	32	D	62	A
3	B	33	B	63	B
4	C	34	C	64	C
5	A	35	A	65	C
6	D	36	C	66	A
7	C	37	D	67	D
8	D	38	D	68	A
9	C	39	D	69	A
10	A	40	C	70	A
11	D	41	C	71	D
12	A	42	B	72	A
13	D	43	A	73	C
14	C	44	A	74	C
15	A	45	A	75	A
16	A	46	B	76	A
17	C	47	A	77	B
18	D	48	B	78	B
19	B	49	C	79	D
20	C	50	A	80	B
21	100	51	10	81	7
22	6	52	60	82	3
23	500	53	10	83	1
24	0.5	54	23.73	84	4
25	400	55	20.24	85	5

See Maths solutions on next page.....

SAFE HANDS & PACE
LT-15 (JEE) Answer key

: ANSWER KEY :

61)	a	62)	a	63)	b	64)	c	82)	3
65)	c	66)	a	67)	d	68)	a	83)	1
69)	a	70)	a	71)	d	72)	a	84)	4
73)	c	74)	c	75)	a	76)	a	85)	5
77)	b	78)	b	79)	d	80)	b		
81)	7								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **((A)**

Any tangent to hyperbola forms a triangle with the asymptotes which has constant area ab

Given

$ab = a^2 \tan\lambda$

$\Rightarrow \dfrac{b}{a} = \tan\lambda$

$\Rightarrow e^2 - 1 = \tan^2\lambda$

$\Rightarrow e^2 = 1 + \tan^2\lambda = \sec^2\lambda$

$\Rightarrow e = \sec\lambda$

62 **((A)**

The given ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Here $a^2 = 16$ and $b^2 = 9$

$\therefore b^2 = a^2(1 - e^2) \Rightarrow 9 = 16(1 - e^2)$

$\Rightarrow e = \dfrac{\sqrt{7}}{4}$

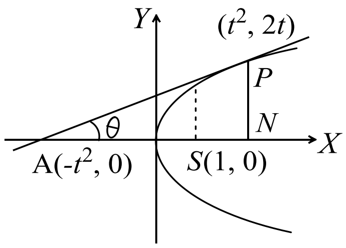
Hence, the foci are $\left(\pm\sqrt{7}, 0\right)$

Radius of the circle=distance between $\left(\pm\sqrt{7}, 0\right)$ and $(0, 3) = \sqrt{7+9} = 4$

63 **((B)**

Tangent at point P is $ty = x + t^2$, when slope of tangent is $\tan\theta = \frac{1}{t}$

Now required area is $A = \frac{1}{2}(AN)(PN) = \frac{1}{2}(2t^2)(2t)$



$A = 2t^3 = 2(t^2)^{3/2}$

Now $t^2 \in [1, 4]$, then $A_{\max}$ occurs when $t^2 = 4$

$\Rightarrow A_{\max} = 16$

64 **((C)**

$(x-3)^2 + (y+1)^2 = (4x+3y)^2$

$\Rightarrow (x-3)^2 + (y+1)^2 = 25\left(\dfrac{4x+3y}{5}\right)^2$

$\Rightarrow PS = 5PM$

$\Rightarrow$ directrix is $4x + 3y = 0$ and focus $(3, -1)$

So equation of transverse axis is $y + 1 = \frac{3}{4}(x - 3)$

$\Rightarrow 3x - 4y = 13$

65 **((C)**

Foci of hyperbola lie on $y = x$. So, the major axis is $y = x$

Major axis of hyperbola bisects the asymptote

$\Rightarrow$ equation of other asymptote is $x = 2y$

$\Rightarrow$ equation of hyperbola is $(y - 2x)(x - 2y) + k = 0$

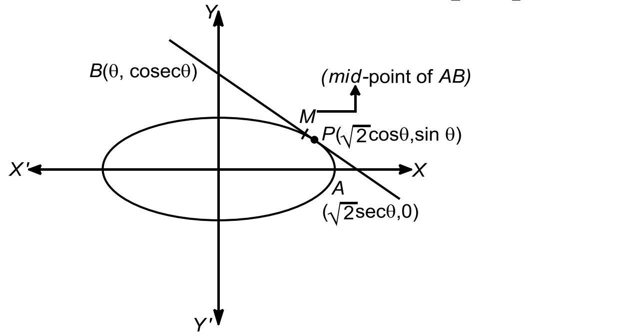
Given that it passes through $(3, 4) \Rightarrow k = 10$

Hence required equation is

$2x^2 + 2y^2 - 5xy + 10 = 0$

66 **((A)**

Let the point $(P\ \sqrt{2}\cos\theta, \sin\theta)$ on $\frac{x^2}{2} + \frac{y^2}{1} = 1$.



Equation of tangent is,

$\frac{x\sqrt{2}}{2}\cos\theta + y\sin\theta = 1$

Whose intercept on coordinate axes are

$A(\sqrt{2}\sec\theta, 0)$ and $B\ (0, cosec\ \theta)$

$\therefore$ Mid-point of its intercept between axes

$\left(\frac{\sqrt{2}}{2}\sec\theta, \frac{1}{2}cosec\ \theta\right) = (h, k)$

$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}h}$ and $\sin\theta = \frac{1}{2k}$

Thus, focus of mid-point M is

$(cos^2\theta + sin^2\theta) = \frac{1}{2h^2} + \frac{1}{4k^2}$

$\Rightarrow \quad \frac{1}{2x^2} + \frac{1}{4y^2} = 1$,is required locus.

67 **((D)**

Ends of latus rectum are $P(a, 2a)$ and $P'(a, 2a)$

Point P has parameter $t_1 = 1$ and point P' has parameter $t_2 = -1$

Normal at point P meets the curve again at point Q whose parameter $t_1' = -t_1 - \frac{2}{t_1} = -3$

Normal at point P' meets the curve again at point Q' whose parameter $t_2' = -t_2 - \frac{2}{t_2} = 3$

Hence, point Q and Q' have coordinates $(9a, -6a)$ and $(9a, 6a)$, respectively

Hence, $QQ' = 12a$

68 **((A)**

Let the given straight line be axis of coordinates and let the equation of the variable line be

$\dfrac{x}{a} + \dfrac{y}{b} = 1$

This line cuts the coordinate axis at the point $A(a, 0)$ and $B(0, b)$

Therefore, the area of ΔAOB is

$\dfrac{1}{2}ab = c^2$

$\Rightarrow ab = 2c^2 \qquad$ (i)

If (h, k) be the coordinates of the middle point of AB, then

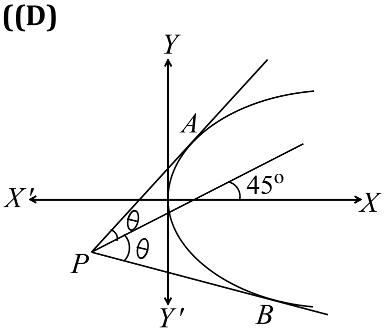
$h = \frac{a}{2}$ and $k = \frac{b}{2} \qquad$ (ii)

On eliminating a and b from Eqns. (i) and (ii), we get
$2hk = c^2$
Hence, the locus of (h, k) is $2xy = c^2$

69 ((A)
According to the question $\frac{2\sqrt{9m^2-49}}{\sqrt{1+m^2}} = 2$
$\Rightarrow 9m^2 - 49 = 1 + m^2$
$\Rightarrow 8m^2 = 50$
$\Rightarrow m = \pm\frac{5}{2}$

70 ((A)
Let (h, k)be the midpoint of the chord $7x + y - 1 = 0$
$\Rightarrow \frac{hx}{1} + \frac{ky}{7} = \frac{h^2}{1} + \frac{k^2}{7}$(i)
And $7x + y = 1$(ii)
Represents same straight line
$\Rightarrow \frac{h}{7} = \frac{k}{7} \Rightarrow h = k$
$\Rightarrow$Equation of the line joining $(0, 0)$ and (h, k) is $y - x = 0$

71 ((D)



Here $\frac{1}{t_1} = \tan\left(\frac{\pi}{4} + \theta\right)$
and $\frac{1}{t_2} = \tan\left(\frac{\pi}{4} - \theta\right)$
So, $t_1 t_2 = 1$
$\Rightarrow$the x-coordinate of $P = at_1t_2 = a$

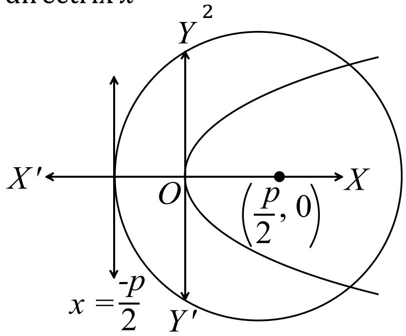
72 ((A)
For $\lambda = -3$, the equation becomes
$x^2 + y^2 - 3xy = 0$
Which represents a pair of lines through origin

73 ((C)
$\left(\sqrt{3h}, \sqrt{3k+2}\right)$ lies on the line $x - y - 1 = 0$
$\Rightarrow \left(\sqrt{3h}\right)^2 = \left(\sqrt{3k+2} + 1\right)^2$
$\Rightarrow 3h = 3k + 2 + 1 + 2\sqrt{3k+2}$
$\Rightarrow 3^2(h - k - 1)^2 = 2^2\left(\sqrt{3k+2}\right)^2$
$\Rightarrow 9(h^2 + k^2 + 1 - 2hk - 2h + 2k) = 4(3k + 2)$
$\Rightarrow 9(x^2 + y^2) - 18xy - 18x + 6y + 1 = 0$
Now $h^2 = ab$ and $\Delta \neq 0$
Therefore, locus is a parabola

74 ((C)
Ellipse passing through $O(0, 0)$ and having foci $P(3,3)$ and $Q(-4, 4)$,
Then $e = \frac{PQ}{OP+OQ}$
$= \frac{\sqrt{50}}{3\sqrt{2} + 4\sqrt{2}}$
$= \frac{5}{7}$

75 ((A)
Given hyperbola is
$\frac{x^2}{1} - \frac{y^2}{\frac{1}{3}} = 1$
Its eccentricity 'e' is given by
$\frac{1}{3} = 1(e^2 - 1)$
Hence, eccentricity e' of the conjugate hyperbola is given by
$1 = \frac{1}{3}(e'^2 - 1)$
$\Rightarrow e'^2 = 4$
$\Rightarrow e' = 2$

76 ((A)
The focus of parabola $y^2 = 2px$ is $\left(\frac{p}{2}, 0\right)$ and directrix $x = -\frac{p}{2}$



$\therefore$ Centre of circle is $\left(\frac{p}{2}, 0\right)$ and radius $= \frac{p}{2} + \frac{p}{2} = p$
$\therefore$ Equation of circle is $\left(x - \frac{p}{2}\right)^2 + y^2 = p^2$
Or $4x^2 + 4y^2 - 4px - 3p^2 = 0$
Solving this circle with the given parabola, we have (eliminating y)
$4x^2 + 8px - 4px - 3p^2 = 0$
$\Rightarrow 4x^2 + 4px - 3p^2 = 0$
$\Rightarrow (2x + 3p)(2x - p) = 0$
$\Rightarrow x = \frac{-3p}{2}, \frac{p}{2}$
$\Rightarrow y^2 = -3p^2$(not possible),
$\Rightarrow y^2 = 2p.\frac{p}{2} \Rightarrow = \pm p$
Therefore, required points are $\left(\frac{p}{2}, p\right), \left(\frac{p}{2}, -p\right)$

77 ((B)
Equation of tangent at (x_1, y_1) is
$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$
It passing through $(0, -b)$. So,
$0 + \frac{y_1}{b} = 1 \Rightarrow y_1 = b$
Equation of normal is
$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2e^2$
Which passes through $\left(2a\sqrt{2}, 0\right)$. Hence,
$\frac{a^2 2a\sqrt{2}}{x_1} = a^2e^2$

$\Rightarrow x_1 = \dfrac{2a\sqrt{2}}{e^2}$

Now (x_1, y_1) lies on the hyperbola

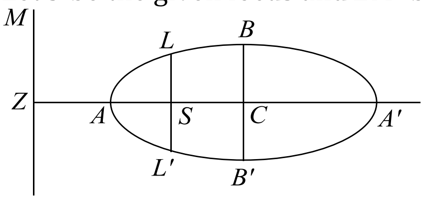
$\dfrac{x_1^2}{a^2} - \dfrac{y_1^2}{b^2} = 1$

$\Rightarrow \dfrac{8}{e^4} - 1 = 1$

$\Rightarrow e^2 = 2$

78 **((B)**

Let S be the given focus and ZM be the given line



Then $SZ = \frac{a}{e} - ae$

$= \dfrac{a}{e}(1 - e^2)$

$= \dfrac{b^2}{ae} = k(\text{say})$

as $b^2 = a^2(1 - e^2)$

Now take SC as x-axis and LSL' as y-axis. Let (x, y) be the coordinates of B with respect to these axes, then $x = SC = ae, y = CB = b$

Hence, $\frac{y^2}{x} = \frac{b^2}{ae} = SZ$, which is constant

$\therefore\ y^2 = kx$ is the required locus which is a parabola

79 **((D)**

As in above question point of intersection is

$(h, k) \equiv \left(\dfrac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}, \dfrac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}\right)$

It is given that $\alpha + \beta = c =$ constant

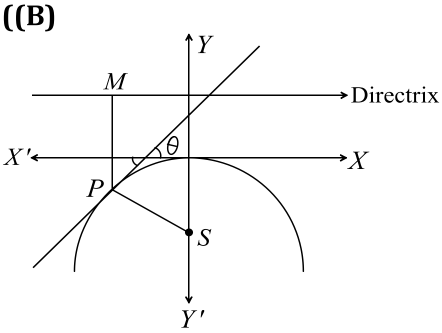
$\Rightarrow h = \dfrac{a\cos\frac{c}{2}}{\cos\left(\frac{\alpha-\beta}{2}\right)}$ and $k = \dfrac{b\sin\frac{c}{2}}{\cos\left(\frac{\alpha-\beta}{2}\right)}$

$\Rightarrow \dfrac{h}{k} = \dfrac{a}{b}\cot\left(\dfrac{c}{2}\right)$

$\Rightarrow k = \dfrac{b}{a}\tan\left(\dfrac{c}{2}\right)h$

$\Rightarrow (h, k)$ lies on the straight line

80 **((B)**



Slope of line $\lambda = \tan\theta$

$\Rightarrow \tan(\angle MPS) = \tan 2\left(\dfrac{\pi}{2} - \theta\right) = \tan(\pi - 2\theta)$

$= -\tan 2\theta = \dfrac{2\lambda}{\lambda^2 - 1}$

Integer Answer Type

81 **(7)**

Here, $(x-1)^2 + (y-3)^2 = \left\{\frac{5x - 12y + 17}{\sqrt{5^2 + (-12)^2}}\right\}^2$

$\therefore$ the focus $= (1,3)$ and the directrix is $5x - 12y + 17 = 0$

The distance of the focus from the directrix

$= \left|\dfrac{5 \times 1 - 12 \times 3 + 17}{\sqrt{5^2 + (-12)^2}}\right| = \dfrac{14}{13}$

$\therefore$ latus rectum $= 2 \times \frac{14}{13} = \frac{28}{13}$

82 **(3)**

Any tangent to parabola $y^2 = 4x, (a = 1)$is

$y = mx + \frac{1}{m}$. It passes through $(-2, -1)$

$\therefore -1 = -2m + \frac{1}{m}$ or $2m^2 - m - 1 = 0$

Or $(2m + 1)(m - 1) = 0$

Or $m = 1/2$ and $m = 1$

Then angle between lines is

$\tan\theta = \left|\dfrac{m_1 + m_2}{1 - m_1 m_2}\right| = 3$

83 **(1)**

Focus of $y^2 = 16x$ is $(4, 0)$

Any focal chord is $y - 0 = m(x - 4)$

Or $mx - y - 4m = 0$

This focal chord touches the circle $(x - 6)^2 + y^2 = 2$

Then distance from the center of circle to this chord is equal to radius of the circle

Or $\frac{|6m - 4m|}{\sqrt{m^2 + 1}} = \sqrt{2}$ or $2m = \sqrt{2}.\sqrt{m^2 + 1}$

Or $2m^2 = m^2 + 1 \Rightarrow m^2 = 1$

$\therefore\ m = \pm 1$

84 **(4)**

$\because OS_1 = ae = 6, OC = b$ (let)

Also $CS_1 = a$

$\because$ Area of $\Delta OCS_1 = \frac{1}{2}(OS_1) \times (OC) = 3b$

$\because$ semi-perimeter of $\Delta OCS_1 = 1/2\ (OS_1 + OC + CS_1)$

$= 1/2(6 + a + b)$ (1)

$\because$ Inradius of $\Delta OCS_1 = 1$

$\Rightarrow \frac{3b}{\frac{1}{2}(6 + a + b)} = 1 \Rightarrow 5b = 6 + a$ (2)

Also $b^2 = a^2 - a^2e^2 = a^2 - 36$ (3)

$\Rightarrow$ from (2)

$25b^2 = 36 + 12a + a^2$

$\therefore 25(a^2 - 36) = 36 + a^2 + 12a$

$2a^2 - a - 78 = 0$

$\therefore a = \dfrac{13}{2}, -6$

$a = \dfrac{13}{2} \therefore b = \dfrac{5}{2}$

85 **(5)**

Points are $A\ (3, 4), B\ (6, 8)$ and $O\ (0, 0)$. $OA + OB = 2a$ (where a is semi-major axis)

$2a = 5 + 10 = 15$

$\therefore\ a = \dfrac{15}{2}$

Now $2ae = \sqrt{(6-3)^2(8-4)^2} = 5$

$e = \frac{1}{3}$

$\therefore\ b^2 = \frac{225}{4}\left(1 - \frac{1}{9}\right) = 50$